

# Concepts of Three-Dimensional Time in Physics

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November 21, 2014

## Abstract.

*Spacetime in physics is represented as 4 variables, with space as a three-dimensional vector and time as a scalar. But what if there is more dimensionality to spacetime, for instance, 3 dimensions of time as well as Scalar Space. Having a location in 3D Vector Time is easy enough to imagine and so is moving in time. When the mathematics of moving in 3D Vector Time is laid out, it forces some new insights into the relationship and symmetry between Space and Time. The idea of three-dimensional time and velocity as a coupler to three-dimensional space is postulated in the Reciprocal System Theory. In this paper, instead of postulates, a mathematical approach is laid out using quaternions to show how the link between 3D Time, 3D Vector Space, Scalar Time and Scalar Space, and how velocity couples all of them.*

## 1.1 Introduction

The concept of location in 3D Vector Space is very familiar to us. So is the concept of moving in 3D Vector Space. But to move in 3D Vector Space requires time. Without time, it is not possible to move in space, since movement is change in space divided by a change in time. A change in space divided by a change in space is not movement.

The concept of 3D Vector Time can be imagined and we can even imagine moving from one location in time to another, just like in 3D Vector Space. But when 3D Vector Time is put into the mathematics, the idea of moving in time becomes an issue. A change in time divided by a change in time is not movement. Just like 3D Vector Space, to move in time requires a change in time and a change in space, specifically a change in time divide by a change in space, or the reciprocal of velocity in 3D Vector Space.

The speed of light is a ratio of space to time, where increasing time has the same effect as decreasing space and vice versa. This reciprocal relationship between space and time will be used to couple space and time in a new way where time is a 3D vector and space is a scalar.

What does 3D Vector Time mean? We intuitively understand 1 dimension of time as a line that goes from the past, through to the present and on into the future. But what about 2 dimensions of time [ $t^2$ ], or even 3 dimensions [ $t^3$ ]. Velocity and reciprocal velocity [ $1/v$ ] will be used to gain understandings of these new dimensions in Vector Time. Dewey Larson, in his Reciprocal System Theory, has raised some very interesting ideas where the concept of 3D Vector Time and the coupling of 3D Vector Time and 3D Vector Space via velocity[1]. In the Reciprocal System Theory, the lack of any mathematical derivation in the Reciprocal System can make it hard to follow how his concepts are derived.

The form used in this paper are quaternions, the same form that Minkowski used to formulate spacetime with 4 dimensions, a 3D vector for space and a scalar form of time. Quaternions were developed by W.R. Hamilton and were used by James Maxwell when he formulated his original Electrodynamics equations. Years after Maxwell's work, Oliver Heaviside found that reducing a 4-dimensional quaternion to a 3-dimensional vector simplified the equations. The rationale for eliminating the scalar component is that the scalar portion had no physical analog, so could be ignored in the interest of simplicity. In this paper, as well as others to follow, the scalar portion has significance. The quaternion also has more complex form

that is known as the bi-quaternion [2]. In this notation, all the scalars and vectors are complex. The bi-quaternion form of the quaternion, although useful in physics, is not explicitly required for the derivations in this paper, and so the standard quaternion is used.

The Minkowski spacetime quaternion is significant because it couples Vector Space and Scalar Time together. It changed the way we looked at the universe. But there is still more to be gained from a reciprocal of this known spacetime quaternion, a new quaternion where the vector is three-dimensional time and the scalar variable is space. These additional 4 dimensions of spacetime and how it applies to electrodynamics is the reason for this paper.

To start off, for clarity, dimensions of space and time will be treated separately in the first two sections and then combined after that.

Note. In this paper, inverse and reciprocal mean the same thing. Three-dimensional Space and Scalar Time will be referred to as 4Space and 3D Time and Scalar Space as 4Space.

## 1.2 3 Dimension Vector Space and Scalar Time [4Space]

In 3D Vector Space and Scalar Time, or 4Space as I will call it from now on, the dimensions of length [L], area [L<sup>2</sup>] and [L<sup>3</sup>] are very familiar to us. These variables define static space. As stated previously, to move from one location in space to another, it cannot be done in space since a change in space relative to space is no change in location and therefore no motion has been achieved.

$$d/dx x = 1, \text{ or } \nabla \mathbf{x} = 3 \text{ where } \mathbf{x} = x_1 \mathbf{i} + x_2 \mathbf{j} + x_3 \mathbf{k} \quad (1)$$

*To reiterate, in order to move in space, a change in time is required, in other words dx/dt or a velocity.* This exercise will be done in detail to elucidate concepts in 3D Space which will be useful when ideas in 3-dimensional time are explored in the next section. Velocity in 1 dimension is represented as

$$d/dt x = dx/dt \quad (2)$$

and in 3 dimensions

$$d/dt \mathbf{x} = \mathbf{v} \text{ where } \mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k} \quad (3)$$

Here any movement is expressed in units of space/time [s/t] or meters/second. It is important to define units in terms of space and time since it will help to understand relationships later.

To get acceleration, again a change in time is necessary.

$$d/dt \mathbf{v} = d\mathbf{v}/dt \text{ or acceleration where } \mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} \quad (4)$$

So to move from one location to another, either at constant velocity or at an ever increasing velocity requires a change in time. Units for acceleration are space/time<sup>2</sup> [s/ t<sup>2</sup>] or meters/second<sup>2</sup>.

### 1.2.1 Space in Quaternion Form

The spacetime quaternion for a location in space is

$$X = (f(x) + \mathbf{i}\cdot\mathbf{x}) \quad \text{where } x \text{ is } x_0 \text{ and } \mathbf{i}\cdot\mathbf{x} \text{ is the complex vector } x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k} \quad (5)$$

where the complex vector satisfies the conditions that  $i^2 = j^2 = k^2 = ijk = -1$ ;  $ij = k$ ;  $jk = i$ ;  $ki = j$  and  $ij = -ji$ ;  $jk = -kj$  and  $ki = -ik$ .

For derivations with quaternions, the operator Nabla  $\nabla$  is used. For this paper, when Nabla is used in reference to 3D Space, I have put a subscript  $s$  on the operator,  $\nabla_s$ . The subscript  $s$  is used to denote that the derivation in vector form is with respect to space. Nabla for space is

$$\nabla_s = (\partial/\partial x_1 + \partial/\partial x_2 + \partial/\partial x_3) \quad (6)$$

In quaternion form, a total derivative  $d/dt$  [3]

$$d/dt = (\partial/\partial t, \mathbf{v}_s \nabla_s) \quad (7)$$

where  $\mathbf{v}_s \nabla_s$  is the vector derivation with respect to time, as is shown below [2]

$$\mathbf{v}_s \nabla_s = + dx/dt \partial/\partial x + dy/dt \partial/\partial y + dz/dt \partial/\partial z = d/dt \quad (8)$$

So, to repeat the exercise of moving in space, the vector derivative of the position quaternion with respect to time is

$$d/dt X = (\partial/\partial t, \mathbf{v}_s \nabla_s)(x + \mathbf{i}\cdot\mathbf{x}) \quad (9)$$

$$V_s = d X/dt = (\partial/\partial t, \mathbf{v}_s \nabla_s X) \text{ or } V_s = (v \ \mathbf{v}_s) \quad (10)$$

Where the quaternion velocity  $V_s$  is in units of space/time or s/t. In free space,

$$V_s = (c \ \mathbf{i}\cdot\mathbf{v}) \quad \text{where } c \text{ is now the speed of light} \quad (11)$$

To finish the exercise, we compute the quaternion for acceleration.

$$A_s = (a + \mathbf{i}\cdot\mathbf{a}) \quad (12)$$

An example of scalar acceleration is gravity, the scalar acceleration of  $9.8 \text{ m/s}^2$ .

### 1.3 3-Dimensional Vector Time and Scalar Space [4Time]

The concept of 3D Vector Time or Scalar Space, or 4Time as I will call it now, is a bit foreign since we are so accustomed to thinking of time in terms of a scalar variable. It can move forward and backwards, but universally we think of time as a scalar that moves forward relentlessly.

To mathematically represent 3D Vector Time is easy. The dimension of time in 1D is [t], in 2D time [t<sup>2</sup>] and in 3D [t<sup>3</sup>]. We have no understanding of what these units mean. Let's start by representing a location in 3D Vector Time. The time vector is represented as follows

$$\mathbf{t} = t_1\mathbf{i} + t_2\mathbf{j} + t_3\mathbf{k} \quad \text{where the axis } i,j,k \text{ are in time} \quad (13)$$

The axis of time in this quaternion still satisfy the same conditions set for quaternion for space in equation 5. Let first go through the same exercise as before, where we start at a location in time and try to move in time. If we apply a delta t to our location in time, we have the same problem as in space

$$d/dt \mathbf{t} = 1, \text{ or } \nabla_t \mathbf{t} = 3 \quad \text{where } \nabla_t = (\partial/\partial t_1 + \partial/\partial t_2 + \partial/\partial t_3) \quad (14)$$

There is no change in the location in time, no motion. In order to move in time, a change in space is required.

$$d/dx \mathbf{t} = dt/dx \text{ or reciprocal velocity} \quad (15)$$

and in 3 dimensions of Vector Space

$$d/ds \mathbf{t} = dt/ds = 1/v_s = \mathbf{v}_t \quad (\text{the reciprocal velocity in 4Time}) \quad (16)$$

So velocity in time, in vector form is represented as

$$\text{Velocity (in time)} = v_{t1} \mathbf{i} + v_{t2} \mathbf{j} + v_{t3} \mathbf{k} \quad (17)$$

The relationship of  $v_t$  to the 4Space velocity is that  $v_t$  equals  $1/v_s$ . So the coefficients in equation 17 could be written as  $1/v_{s1}$ ,  $1/v_{s2}$  and  $1/v_{s3}$ . The units would be time/space or t/s [seconds/meter]. So in 4Time, movement is done in reciprocal velocity relative to our common experience of velocity in 4Space. To finish up our derivation of a change in 4Time, we now determine how to move faster between locations in 4Time.

To have a change in velocity, or acceleration, another change in space is required.

$$d/ds \mathbf{v}_t = d\mathbf{v}_t/ds = \mathbf{a}_t \quad \text{where } \mathbf{a}_t = a_{t1} \mathbf{i} + a_{t2} \mathbf{j} + a_{t3} \mathbf{k} \quad (18)$$

The units for acceleration in 3D Vector Time are t/s<sup>2</sup> [seconds/meter<sup>2</sup>]. To complete the exercise, we complete quaternions for location, velocity and acceleration in 4Time.

### 1.3.1 Time in Quaternion form

So quaternion form of a location in 4Time is

$$T = (f(t) + \mathbf{i} \cdot \mathbf{t}) \quad \text{where } t \text{ is } t_0 \text{ and } \mathbf{i} \cdot \mathbf{t} \text{ is } t_1 \mathbf{i} + t_2 \mathbf{j} + t_3 \mathbf{k} \quad (19)$$

where  $f(t)$  is a scalar function of time and  $\mathbf{t}$  the vector to a location in time.

To move in 4Time, we need a new definition of the total derivative for 4Time with respect to space.

$$d/ds = (\partial/\partial s, (\mathbf{1}/v_s) \nabla_t) = (\partial/\partial s, \mathbf{v}_t \nabla_t) \quad (20)$$

which is the reciprocal of the total derivative  $d/dt$  shown in equation 7.

$$(\mathbf{v}_t) \nabla_t = d t_1/ds \partial/\partial t_1 + d t_2/ds \partial/\partial t_2 + d t_3/ds \partial/\partial t_3 = d/ds \quad (21)$$

So, quaternion Velocity in 4Time is

$$d/ds T = (\partial/\partial s, (\mathbf{v}_t) \nabla_t)(f(t), \mathbf{i} \cdot \mathbf{t}) \quad (22)$$

$$d/ds T = ((\partial f(t)/\partial s, (\mathbf{v}_t) d/ds(\mathbf{i} \cdot \mathbf{t})) = (v \mathbf{i} \cdot \mathbf{v}_t) = \nabla_t \quad (23)$$

where  $v$  is speed, likely  $c$ , and  $\mathbf{v}_t$  is velocity in three-dimensional time,  $\nabla_t$  is the quaternion for velocity in 4Time. The quaternion acceleration  $\Delta_t$  in 4Time is

$$\Delta_t = (a + \mathbf{i} \cdot \mathbf{a}_t) \quad (24)$$

### 1.4 How to combine it all.

So far space and time have been evaluated separately, to clarify the concepts involved in 3 dimensional time. Physics has space and time coupled as shown in the previous sections. In order to move in spacetime now requires either  $dx/dt$  or  $dt/dx$ . The two quaternions,  $\mathbb{X}$  and  $\mathbb{T}$ , need to be coupled. These two coupled 4 dimensional representations of spacetime will result in a total of 8 spacetime variables.

In the Minkowski representation of spacetime in physics, there a Scalar Time and a Vector Space.

$$\mathbb{X}(t) = (c f(t) + \mathbf{i} \cdot \mathbf{x}) \quad \text{where } f(t) \text{ is a Scalar Time field.} \quad (25)$$

There are mathematical operators that allow us to go from scalar to vector and vector to scalar. The mathematical operation divergence, represented here as  $\nabla_s \cdot$  and  $\nabla_t \cdot$ , will give us a scalar field value from a space vector in the former and a time vector in the latter. The result of this divergence operator will determine if the scalar field is a source or a sink. If Scalar Time is a source in 3D Space, then in order to conserve energy in all these dimensions, Scalar Space would have to be a sink in 3D Time, or vice versa.

The gradient operators,  $\nabla_s$  and  $\nabla_t$ , give a vector from a scalar. So if we apply the time gradient operator to the time scalar field value of equation 25, where  $t$  is a function of Scalar Time a vector form of  $\mathbf{t}$  is the result.

$$\nabla_t(f(t)/c) = \mathbf{i} \cdot \mathbf{t} \quad \text{where } \nabla_t \text{ is the gradient with respect to time} \quad (26)$$

If the space divergence operator is applied to the space vector in equation 25, scalar field of space is calculated. The results for scalar field space is shown next, where  $\mathbf{v}$  equals the speed of light

$$\nabla_s \cdot \mathbf{i} \cdot \mathbf{x} = f(s) \quad \text{where } f(s) \text{ is a Scalar Space field.} \quad (27)$$

Combing the results the quaternion representation of spacetime for 4Time is

$$T = (f(s)/c, \mathbf{i} \cdot \mathbf{t}) \quad (28)$$

The two quaternions are coupled via divergence and gradient, as shown in the next equation.

$$T(\mathbf{s}, \mathbf{t}) = ((\nabla_s \cdot X(\mathbf{t}, \mathbf{s})) + \mathbf{i} \cdot (\nabla_t X(\mathbf{t}, \mathbf{s}))) = (f(s)/c + \mathbf{i} \cdot \mathbf{t}) \quad (29)$$

and

$$X(\mathbf{t}, \mathbf{s}) = ((\nabla_t \cdot T(\mathbf{s}, \mathbf{t})) + \mathbf{i} \cdot (\nabla_s T(\mathbf{s}, \mathbf{t}))) = (c f(t) + \mathbf{i} \cdot \mathbf{x}) \quad (30)$$

From there we can take the total derivative with respect to space to get velocity and acceleration in time.

## 1.5 Implications for Speed of Light

In relativity, the interval in spacetime is invariant, so this property will be used. In a moving frame, where an event is measured at the same place, then  $x' = 0$ , and the equation for the interval of time in the moving frame becomes,

$$t' = (t^2 - x^2)^{1/2} \quad (31)$$

where in 4Space, Scalar Time in terms of Vector Space [meters] is

$$c t = x \quad \text{units of meters} \quad (32)$$

The derivation for  $t'$  is done in the Appendices and the result is shown below,

$$t' = t(1 - v^2/c^2)^{1/2} = t \gamma_s \quad (33)$$

and

$$\gamma_s = 1/(1 - v^2/c^2)^{1/2} \quad (34)$$

where  $v < c$  to keep  $t'$  from going negative, so the velocity limit is a *max of c*.

For spacetime effects in 4Time, the velocity of light is  $dt/dx$  in 4Time, so Scalar Space in terms of components of Vector Time are

$$x/c = \text{units of seconds} \quad (35)$$

and the transformation of the Vector Time field is,

$$t' = t / \gamma_t \quad (36)$$

A new Lorentz contraction factor, labelled  $\gamma_t$  has to be calculated. This new form for  $\gamma_t$  as well as the transformation of time in 4Space are derived in the Appendices. The new form for  $\gamma_t$  is

$$\gamma_t = 1/(1 - c^2/v^2)^{1/2} \text{ where } v > c \text{ to keep } \gamma_t \text{ from going negative.} \quad (37)$$

The term  $v^2/c^2$  in  $\gamma_t$  means that

$$v^2/c^2 > 1 \text{ so } v \text{ always is greater than } c \quad (38)$$

The most important result is that in 4Time, Vector Time and Scalar Space, the velocity is always superluminal. The variable  $t$  in Vector Time is one component of a vector field, unlike in 4Space, where it is the only component of the Scalar Time field.

## 1.6 Velocity as a coupling

So far, there are two spacetime quaternions. The first one,  $X(t, \mathbf{s})$ , couples Scalar Time field and Vector Space field. A scalar field is defined as a field for which there is a single value for each point in the field. The vector field has a magnitude and direction for each point in the field. The second one,  $T(\mathbf{s}, \mathbf{t})$ , couples Scalar Space field and Vector Time field. Locations in 4Space  $[X(t, \mathbf{s})]$  are in units of space, or  $s$ . In 4Time  $[T(\mathbf{s}, \mathbf{t})]$ , locations in time are in seconds which will be represented as  $t$  for time. The reason for

this unit representation is so velocity can be represented for what it is, space/time, or s/t. To convert a location in 4Space using time, velocity is needed to keep the units in space, as in,  $ct$ . In 4Time,  $X(t, \mathbf{s})$  is multiplied by the  $1/c$  to keep the units in time, so,  $x/c$ .

$$(\text{Location in}) \text{ time} * (c) \text{ space/time} = (\text{Location in}) \text{ space} \rightarrow t * s/t = s \quad (39)$$

$$(\text{Location in}) \text{ space} * (1/v) \text{ time/space} = (\text{Location in}) \text{ time} \rightarrow s * t/s = t \quad (40)$$

Previously, it was just a location in spacetime. Now there is movement in spacetime, or velocity. To change from velocity in 4Space to velocity in 4Time, the reciprocal velocity squared is required.

$$\text{velocity (in space)} \times 1/v^2 = \text{reciprocal velocity (in time)} \quad (41)$$

Looking at the coupling in units, it gives a better insight

$$s/t * t^2/s^2 = t/s \quad \text{where } t/s \text{ is reciprocal velocity in 4Time} \quad (42)$$

For accelerations, reciprocal velocity cubed is required. If we take equation 41 and take the time derivative of velocity, we get acceleration [units  $s/t^2$ ]. The space derivative of  $t/s$ , that is,  $d/ds t/s$  is equal to  $t/s^2$  and the coupling between the two of reciprocal velocity cubed. The equation becomes,

$$s/t^2 * t^3/s^3 = t/s^2 \quad (43)$$

***The most important point of this section is that Scalar Time and Vector Space [4Space] and Scalar Space and Vector Time [4Time] are coupled via velocity and reciprocal velocity to powers of 1, 2 and 3. This coupling is crucial for the next section and for future understanding.***

Dewey Larson postulated that  $t^3/s^3$ , or  $1/c^3$  is mass. The term  $s/t^2$  is the familiar acceleration in Vector Space. The idea of not treating mass as a physical property was raised in a number of papers on mass, inertia and ZPF. In one particular paper, two particular statements are very important. The first statement is, "In our formulation, the  $m$  in Newton's second law of motion,  $F=ma$ , becomes nothing more than a coupling constant between acceleration and an external electromagnetic force." The second statement deal with another famous equation,  $E=mc^2$  and states "Mass is energy" [5]. With the middle term of equation 43,  $t^3/s^3$ , equivalent to mass and used as a coupling term, this implies that the term  $t/s^2$  is force, in terms of Vector Time. With force equal to  $t/s^2$ , then energy is force times distance, or

$$\text{time/space}^2 \times \text{space [distance]} = \text{time/space or } 1/c \quad \text{in units of } t/s \quad (44)$$

Energy then is equivalent to  $t/s$ , or inverse velocity  $1/c$ . Using the equation  $E=mc^2$  also confirms the same conclusion that energy is equal to  $1/c$ . This means if the total derivative,  $d/ds$ , of any location in Vector Time, it is equivalent to Vector Energy. The quaternion representation for energy is

$$d/ds T = (e + \mathbf{E}) \quad \mathbf{E} = iE_{t1} + jE_{t2} + kE_{t3} \quad \text{in units of } t/s \quad (45)$$



A Scalar Energy is familiar in 4Space, but a Vector Energy is a new concept. The Scalar Energy is the portion of motion in 3D Time that shows up as a scalar in 3D Space. The divergence operator compresses this 3-dimensional motion in into a scalar form in 3D Space.

If the total derivative  $[d^2/ds^2]$  is taken twice, just like acceleration  $[d^2/dt^2]$  in 4Space, then the quaternion representation is

$$d^2/ds^2 T = (f + \mathbf{F}) \quad \mathbf{F} = iF_{t1} + jF_{t2} + kF_{t3} \quad \text{in units of } t/s^2 \quad (46)$$

In this equation, it is Vector Force that is familiar in Vector Space and Scalar Force that is not. In this case, the Scalar Force should always equal zero, since a uniformly diverging or converging force field sums to zero. When the scalar field is not uniformly diverging or converging, then a Scalar Force, like gravity, will result.

## 1.7 Summary

Movement in time manifests as energy and acceleration in time manifests as a force. The familiar Scalar Time field is 3D Vector Time compressed into 3D Scalar Time. This scalar field must have extremely high uniformity since Scalar Time is used as a one-dimensional scalar variable without affecting the physics. It appears we have much more to learn about time. 3D Vector Time is easier to imagine, but movement in Vector Time requires Scalar Space, resulting in a reciprocal velocity from the perspective of 3D Vector Space. Just like the Scalar Time field manifests as a uniform scalar field, that is, as a uniform density in 3D Space, so it could be imagined that Scalar Space field is a uniform density in 3D Vector Time.

With so many extra degrees of freedom, it helps to keep an open mind and explore what the implication can be. It is clear that Physics does constrain time to 1 dimension. The extra equations of physics that come from time in 3 dimensions and space as a scalar variable open a whole new way of looking at the interactions of space and time.

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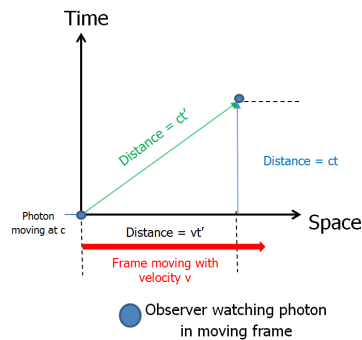
## Version

1. Nov 21, 2014. Original.
2. Dec 29, 2014. Added in reference 5 and made reference to it in section 1.6.
3. Jan 02, 2015. Corrected typo in c term for equations 79 and 80
4. Jan 17, 2015. Add in vector in 3D Time to differentiate it from 3D Scalar Time field.
5. Nov 30, 2015. In conclusion, add description of electric space and magnetic time.
6. Dec 4, 2016. Added Appendics and changed section 1.6. Added Changed email address.
7. Dec 30, 2017. Reword Abstract, add complex quaternion relations in equation 5, update terminology so 4DS is 4Space and 4DT is 4Time since Scalar Time and Scalar Space are not dimensions.
8. Mar 11, 2018. Delete all the sections on electromagnetic equations and make this paper only about space, time and quaternions.

## Appendix A: Calculation of gamma for Vector Space and Vector Time

First, let's use the technique to calculate  $\gamma_s$ . Set a frame for the moving particle moving up and down in the same location inside the moving frame and one for an observer watching the particle outside the moving frame, where the frame is moving at velocity  $v$ . In the particle frame, the particle would move up and down in the same location, moving along the time axis for a distance  $ct$ . The observer sees the particle move along the green line, for a distance  $ct'$ . The frame moves a distance  $vt'$  in this time.

Laying out the geometry, we can determine how much change in space, or time, there need to be to make the motion the same for both frames. This result is  $\gamma_s$  for Vector Space and Scalar Time.



$$(ct')^2 = (vt')^2 + (ct)^2$$

$$c^2 t'^2 = t'^2 (c^2 - v^2)$$

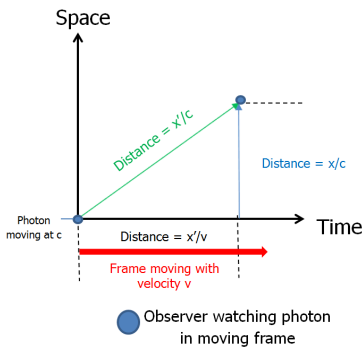
$$t = t' (1 - v^2/c^2)^{1/2}$$

$$t' = 1/(1 - v^2/c^2)^{1/2} t$$

$$t' = \gamma_s t$$

$$\text{where } \gamma_s = 1/(1 - v^2/c^2)^{1/2}$$

Now for Vector Time and Scalar Space



$$(x'/c)^2 = (x/c)^2 + (x'/v)^2$$

$$x^2/c^2 = x'^2 (1/c^2 - 1/v^2)$$

$$x = x'(1 - c^2/v^2)^{1/2}$$

$$x' = 1/(1 - c^2/v^2)^{1/2} x$$

$$x' = \gamma_t x$$

where  $\gamma_t = 1/(1 - c^2/v^2)^{1/2}$

## Appendix B: Calculation of Lorentz contraction factor for Vector Time

Now in Vector Time, Time is the length of the Vector Field and Space is the progression of the Scalar Field.

$$t_0 = t_2 - t_1$$

$$t' = t'_2 - t'_1$$

$$t = \gamma_t (t' - x/v')$$

put  $t_0$  and  $t$  into equation above and subtracting them gives

$$t_2 - t_1 = \gamma_t ((t'_2 - x'_2/v) - (t'_1 - x'_1/v))$$

now  $x'_1 = x'_2$  because this happens at the same location in the moving frame. These terms cancel

$$t_2 - t_1 = \gamma_t (t'_2 - t'_1)$$

or

$$t' = t_0 / \gamma_t$$

This correlates well with the form of the Vector Space field under motion, where

$$L = L_0 / \gamma_s$$

The behavior of  $\gamma_t$  and  $\gamma_s$  make the behavior different, but the form is the same.

The form for the behavior of the scalar field is also the same. For the Scalar Time field, it is

$$t' = \gamma_s t_0$$

and for the Scalar Space field associated with the Vector Time field, it is

$$x' = \gamma_t x_0$$