Electrodynamic Potentials in Three-Dimensional Time

Robert Kersten
robert@multidimensionaltime.com

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Abstract.

Spacetime in physics is represented as a four-dimensional vector, with time as a scalar and space as a three-dimensional vector. But as shown in the paper 'Concepts of Three Dimensional Time in Electrodynamics,' if a time domain is included where time is a three-dimensional vector in time and space is scalar, there are a total of eight equations for electrodynamics. In this paper, the well-known derivation of Maxwell's equations in terms of scalar and vector potentials is shown, and then the same approach is used to derive scalar and vector potentials for 3D Vector Time and Scalar Space. The exercise of having all eight equations in terms of scalar and vector potential shows that the scalar wave equation is exactly the same for familiar space domain and also for this new time domain. The significance is that the Bohm Aharanov effect demonstrates that potentials have an effect on matter just as magnetic and electric fields do.

1.1 Introduction

In this paper, the electrodynamic potentials are evaluated with regards to the familiar 3D Vector Space and Scalar Time as well as 3D Vector Time and Scalar Space. The Bohm Aharanov effect demonstrates that vector potentials have real effects on electrons, shifting their phase. This means that potentials have effect on matter and should be included in the analysis of electrodynamics in three-dimensional vector time.

The concept of location in 3D Vector Space is very familiar to us. So is the concept of moving in 3D Vector Space. But to move in 3D Vector Space requires time. Without at least one dimension of time, it is not possible to move in space, since movement is change in space divided by change in time. A change in space divided by a change in space is not movement.

The concept of 3D Vector Time can be imagined, and we can even imagine moving from one location in time to another, just like in 3D Vector Space. But when 3D Vector Time is put into the mathematics, the idea of moving in time becomes an issue. A change in time divided by a change in time is not movement. Just like 3D Vector Space, to move in time requires a change in time and a change in space, specifically a change in time divided by a change in space, or the reciprocal of velocity in 3D Vector Space.

The speed of light is a ratio of space to time; increasing time has the same effect as decreasing space and vice versa. This reciprocal relationship between space and time will be used to couple space and time in a new way, where time is a three-dimensional vector and space is a scalar.

Note: In this paper, inverse and reciprocal mean the same thing. Also, Vector Space and Scalar Time will be referred to as 4Space and Vector Time and Scalar Space as 4Time.

1.1 Maxwell's equations in 3D Vector Time with SI units

For Maxwell's equations in SI units, the units of the electric field are now V/m and magnetic flux is V*s/m^2. Permittivity (ε_0) is in units of A*s/V*m and permeability(μ_0) is in units of V*s/A*m, where and ε_0 times μ_0 equals $1/c^2$.

$\nabla_s \cdot E_s = - \rho_{es}/\varepsilon_0$ units of V/m^2 (1)

$\nabla_s \cdot B_s = 0$ units of Wb/m^3 or V*s/ m^3 (2)

$\nabla_s \times E_s + \partial B/\partial t = 0$ units of V/m^2 (3)

$\nabla_s \times B_s = \mu_0 J_{es} + 1/c^2 \partial E/\partial t$ units of Wb/m^3 (4)

Using the new degrees of freedom, the proposed form of Maxwell's equations for 4Time and 4Space are the following: Equations relating to the electric field charge density and equation relating to magnetic field [B] have flux density.

For SI units, the following relations between Vector Space and Vector Time are important. This relationship is the only way the units work out and what the units suggest is very important.

$E_t = c^2 E_s$ units of Volt*meter/second^2 (5)

In Vector Time [4Time], the electric field in time is related to the electric field in space by the relationship $E_t = c^2 E_s$ which converts V/m to V*m/s^2. This changes the electric field from one based on charge density to an electric flux.

$B_t = c^2 B_s$ units of Volts/second (6)

It is important to remember that since these next four equations describe electrodynamics in 3D Time, the basis units is per second instead of per meter as in 4Space, since volume is measured in seconds instead of meters. Any movement in the scalar direction is time in 3D Space and space in 3D Time. The curl is taken with respect to time, so the space derivate is taken for the variable in the scalar dimension. Since magnetic monopoles are in 3D Vector Time, the units for the magnetic field are V/s^2, that is charge density, and the units for the electric field are in terms of flux so the units are V*m/s^2. In 3D Time, Maxwell's equations become,

$\nabla_t \cdot E_t = 0$ units of V*m/s^3 (7)

$\nabla_t \cdot B_t = - \mu_0 \rho_{mt}$ units are V/s^2 (8)

$\nabla_t \times E_t - c^2 \partial B_t /\partial s = J_{mt}/ \varepsilon_0$ units are V*m/s^3 (9)

$\nabla_t \times B_t + \partial E_t / \partial s = 0$ units of V/s^2 (10)

When magnetic charge density, or magnetic monopoles, along with an electrical flux, are introduced in 3D Vector Time, a symmetry is created with the electrical charge density and magnetic flux in 3D Space. In 3D Vector Space, ε_0 is the electrical charge term, since electric fields have a source in charges and u_0 is the magnetic flux. In 3D Vector Time, Electric fields are not derived from monopoles charges, and so E fields are treated as flux so ε_0 is the electrical flux term and u_0 the magnetic charge term.

Electric monopoles are measurable in 4Space. But magnetic monopoles are not, just like three-dimensional vector time is not yet measurable. From these derivations, once one is detected, the other will also be detectable.

These eight electrodynamic equations give the full symmetry between space and time, impacting how electric and magnetic fields relate to these extra dimensions of spacetime. These additional degrees of freedom, as radical as they might currently appear, allow for magnetic monopoles from 3D Vector Time and Scalar Space, where motion is always faster than light.

Movement in time manifests as energy, and acceleration in time manifests as a force. The familiar Scalar Time field is 3D Vector Time compressed into 3D Scalar Time. This scalar field must have extremely high uniformity since Scalar Time is used as a one-dimensional scalar variable without affecting the physics. It appears we have much more to learn about time. 3D Vector Time is easier to imagine, but movement in Vector Time requires Scalar Space, resulting in a reciprocal velocity from the perspective of 3D Vector Space. Just like the Scalar Time field manifests as a uniform scalar field, that is, as a uniform density in 3D Space, so it could be imagined that Scalar Space field is a uniform density in 3D Vector Time.

1.2 Vector and Scalar Potentials in 3D Vector Space and Scalar Time.

To derive the electrodynamic equations from potentials is done by starting with the equations[1]

$\nabla_s \cdot E_s = -\rho_{es}/\varepsilon_0$ units of V/m^2 (11)

$\nabla_s \cdot B_s = 0$ units of Wb/m^3 or $V*s/m^3$ (12)

$\nabla_s \times E_s = -\partial B_s / \partial t$ units of V/m^2 (13)

$\nabla_s \times B_s = \mu_0 J_{es} + 1/c^2 \, \partial E / \partial t$ units of Wb/m^3 (14)

Since $\nabla_s \cdot B_s = 0$, it must be the curl of some other vector. In electrodynamics, the vector chosen is A, which is the magnetic vector potential. It is represented as

$B_s = \nabla_s \times A_s$ [B field in terms of a potential] (15)

From electrostatics that know that $\nabla_s \times E_s$ is always 0, so the E field can be represented as the gradient of some scalar function. The E field is represented as

$E_s = -\nabla_s \Phi$ (16)

The first equation used to derive potentials is the curl of E with the partial derivative of B with respect to time replace by equation 15. This section follows the derivation done by Richard Feynman [2].

$$\nabla_s \times E_s + \partial/\partial t\,(\nabla_s \times A_s) = 0 \qquad (17)$$

$$\nabla_s \times (E_s + \partial/\partial t\, A_s) = 0 \qquad (18)$$

The curl of a vector function that equals zero can be derived from the gradient of a scalar function,
In this case, the vector is set to the scalar function Φ. The same one in equation 16 is used,

$$E_s + \partial/\partial t\, A_s = -\nabla_s \Phi \qquad (19)$$

Now the electric field can be written in terms of the scalar and vector potentials.

$$E_s = -\nabla_s \Phi - \partial/\partial t\, A_s \text{ [E field in terms of potentials]} \qquad (20)$$

In order to define both the electric field and magnetic field in terms of potentials, we need four equations for the potentials.

It is possible to define a new potential function that does not change the physics of the E and B fields.

$$A' = A + \nabla_s \chi \qquad (21)$$

and

$$\Phi' = \Phi - \partial/\partial t\, \chi \qquad (22)$$

Now the objective is to determine A and Φ from the equations with ρ and J. Using the divergence of the electric field

$$\nabla_s \cdot E_s = -\rho_{es}/\varepsilon_0 \qquad (23)$$

and using the E field from equation 20

$$\nabla_s \cdot (-\nabla_s \Phi - \partial/\partial t\, A_s) = -\rho_{es}/\varepsilon_0 \qquad (24)$$

$$\nabla^2_s \Phi - \partial/\partial t\,(\nabla_s \cdot A_s) = -\rho_{es}/\varepsilon_0 \qquad (25)$$

So we have charge density in terms of scalar and vector potential, but it is not complete because there is still the divergence of A term.

Using the last Maxwell equation 14

$$\nabla_s \times B_s - 1/c^2\, \partial E/\partial t = \mu_0 J_{es} \qquad (26)$$

And substitute in the magnetic vector potential and E from equation 20

$$\nabla_s \times (\nabla_s \times A) - 1/c^2\ \partial/\partial t\ (- \nabla_s\Phi - \partial/\partial t\ A_s) = \mu_0 J_{es} \quad (27)$$

Using the vector identity

$$\nabla_s \times (\nabla_s \times A) = \nabla_s (\nabla_s \cdot A) - \nabla^2_s A \quad (28)$$

and applying this vector identity into equation 27

$$-\nabla^2_s A + \nabla_s (\nabla_s \cdot A) + 1/c^2\ \partial/\partial t\ \nabla_s\Phi + 1/c^2\ \partial^2/\partial t^2\ A = \mu_0 J_{es} \quad (29)$$

Again, there is the option to choose an arbitrary divergence of A, which still satisfies the physics of E and B.

$$\nabla_s \cdot A = - 1/c^2\ \partial/\partial t\ \Phi \quad (30)$$

Inserting equation 30 into 29

$$-\nabla^2_s A - \cancel{\nabla_s (1/c^2\ \partial/\partial t\ \Phi)} + \cancel{1/c^2\ \partial/\partial t\ \nabla_s\Phi} + 1/c^2\ \partial^2/\partial t^2\ A = \mu_0 J_{es} \quad (31)$$

The second and third term cancels, so the equation simplifies to

$$\nabla^2_s A - 1/ c^2 \partial^2/\partial t^2\ A = - \mu_0 J_{es} \quad (32)$$

Now we go back and insert the definition of the divergence of A into equation 25:

$$\nabla^2_s \Phi - 1/c^2\ \partial^2/\partial t^2\ \Phi = - \rho_{es}/\varepsilon_0 \quad (33)$$

So now all Maxwell's equations defined in terms of scalar and vector potentials

$$E_s = - \nabla_s\Phi - \partial/\partial t\ A_s \quad (34)$$

$$B_s = \nabla_s \times A_s \quad (35)$$

$$\nabla^2_s \Phi - 1/c^2\ \partial^2/\partial t^2\ \Phi = - \rho_{es}/\varepsilon_0 \quad (36)$$

$$\nabla^2_s A - 1/ c^2\ \partial^2/\partial t^2\ A = - \mu_0 J_{es} \quad (37)$$

In free space, $\rho_{es} = 0$, equation 36 becomes the scalar wave equation

$$\partial^2/\partial x^2\ \Phi = 1/c^2\ \partial^2/\partial t^2\ \Phi$$

(38)

1.3 Vector and Scalar Potentials in 3D Vector Time and Scalar Space

In Vector Time and Scalar Space, the electric field is a flux, so is derived from an Electric vector potential F_t and the magnetic field is derived from a scalar potential ψ_t. In Vector Space, the units of scalar potential is volts and the magnetic vector potential is V*s/m. In Vector Time, the scalar potentials is still volts, but the electric vector potential becomes V*m/s.

$$E_t = - \nabla_t \times F_t \quad \text{units of } V{*}m/s^2 \qquad (39)$$

and

$$B_t = \nabla_t \psi_t \quad \text{units are } V/s^2 \qquad (40)$$

In Electrodynamics, the equation for the electric field and the electric vector potential, F, is defined as $E_s = \nabla_s \times F$. For the magnetic field and the scalar magnetic potential ψ, the equation is $B_s = \nabla_s \psi$. But these do not work for 3D Vector Time because the del operator for 3D Time is different than for 3D Space and the units are in terms of time for area and volume and time/space for motion. In the space domain, the vector potential A_s has units of V*s/m, but in the time domain, F_t has units of V*m/s. The units for Φ and ψ_t are the same, both V.

The starting electrodynamic equations for 3D time are [1],

$$\nabla_t \cdot E_t = 0 \quad \text{units of } V{*}m/s^3 \qquad (41)$$

$$\nabla_t \cdot B_t = \mu_0 \rho_{mt} \quad \text{units are } V/s^2 \qquad (42)$$

$$\nabla_t \times E_t - c^2 \partial B_t /\partial s = J_{mt}/ \varepsilon_0 \quad \text{units are } V{*}m/s^3 \qquad (43)$$

$$\nabla_t \times B_t + \partial E_t /\partial s = 0 \quad \text{units of } V/s^2 \qquad (44)$$

Beginning with equation 44 and inserting equation 39, the results are

$$\nabla_t \times B_t + \partial /\partial s \, (-\nabla_t \times F_t) = 0 \qquad (45)$$

$$\nabla_t \times (B_t - \partial /\partial s \, F_t) = 0 \qquad (46)$$

As before, a vector function that has a curl of zero can be defined as the gradient of a scalar function. Using the same logic the previous section,

$$B_t - \partial /\partial s \, F_t = -\nabla_t \psi_t \qquad (47)$$

Rearranging this equation,

$$B_t = -\nabla_t \psi_t + \partial /\partial s \, F_t \qquad (48)$$

Again, redefining the vector and scalar potentials so they do not change the physics of the E and B fields.

$$F_t' = F_t + \nabla_t \chi \tag{49}$$

$$\psi_t' = \psi_t - \partial/\partial s\, \chi \tag{50}$$

$$\nabla_t \cdot B_t = \mu_0\, \rho_{mt} \tag{51}$$

$$-\nabla_t \cdot (\nabla_t \psi_t - \partial/\partial s\, F_t) = \mu_0\, \rho_{mt} \tag{52}$$

$$\nabla^2_t\, \psi_t - \partial/\partial s\, (\nabla_t \cdot F_t) = \mu_0\, \rho_{mt} \tag{53}$$

To complete the derivation, we need to solve the electrodynamic equation 43 in time.

$$\nabla_t \times E_t - c^2\, \partial B_t/\partial s = J_{mt}/\varepsilon_0 \tag{54}$$

And substitute in equations for E_t and B_t

$$\nabla_t \times (-\nabla_t \times F_t) + c^2\, \partial/\partial s \nabla_t \psi_t + c^2\, \partial^2/\partial s^2\, F_t = J_{mt}/\varepsilon_0 \tag{55}$$

Using the vector identity

$$\nabla_t \times (-\nabla_t \times F) = -\nabla_t (\nabla_t \cdot F) + \nabla^2_t F \tag{56}$$

Applying this vector identity in equation 55

$$\nabla^2_t F_t - \nabla_t (\nabla_t \cdot F) - c^2\, \partial/\partial s\, \nabla_t \psi_t - c^2\, \partial^2/\partial s^2\, F_t = J_{mt}/\varepsilon_0 \tag{57}$$

Again, there is the option to choose an arbitrary divergence of A, which still satisfies the physics of E and B.

$$\nabla_t \cdot F_t = -c^2\, \partial/\partial s\, \psi_t \tag{58}$$

Now insert equation 58 into 57

$$\nabla^2_t F_t + \cancel{\nabla_t (c\, \partial/\partial s\, \psi_t)} - \cancel{c\, \partial/\partial s\, \nabla_t \psi_t} + c^2\, \partial^2/\partial s^2\, F_t = J_{mt}/\varepsilon_0 \tag{59}$$

The second and third term cancel, so the equation simplifies to

$$\nabla^2_t F_t + c^2\, \partial^2/\partial s^2\, F_t = J_{mt}/\varepsilon_0 \tag{60}$$

Now we go back and insert the definition of the divergence of F_t into equation 53.

$$\nabla^2_t \cdot \psi_t - \partial/\partial s\, (c^2\, \partial/\partial s\, \psi_t) = \mu_0\, \rho_{mt} \tag{61}$$

Which simplifies to

$$\nabla^2_t \cdot \psi_t - c^2 \partial^2/\partial s^2 \psi_t = \mu_0 \rho_{mt} \quad (62)$$

So now we have all the equation defined in terms of scalar and vector potentials.

$$E_t = - \nabla_t \times F_t \quad (63)$$

$$B_t = -\nabla_t \psi_t + \partial/\partial s\, F_t \quad (64)$$

$$\nabla^2_t \cdot \psi_t - c^2 \partial^2/\partial s^2 \psi_t = \mu_0 \rho_{mt} \quad (65)$$

$$\nabla^2_t F_t + c^2 \partial^2/\partial s^2 F_t = J_{mt}/ \varepsilon_0 \quad (66)$$

1.4 Wave equation

In free space, the wave equation for 4Space is shown to be

$$\partial^2/\partial x^2 \Phi = 1/c^2 \partial^2/\partial t^2 \Phi \quad (67)$$

In free time, $\rho_{mt} = 0$, so equation 65 becomes the scalar wave equation for 4Time,

$$\partial^2/\partial t^2 \psi_t = c^2 \partial^2/\partial x^2 \psi_t \quad (68)$$

Rearranging this equation

$$\partial^2/\partial x^2 \psi_t = 1/c^2 \partial^2/\partial t^2 \psi_t \quad (69)$$

Equations 67 and 69 are the same equation since Φ and ψ_t have the exact same units, Volts. This seems to suggest that potentials are common to both 4Space and 4Time.

For the vector potential in 4Space, it is

$$\partial^2/\partial x^2 A_s = 1/c^2 \partial^2/\partial t^2 A_s \quad (70)$$

For 4Time, it can be rearranged to be

$$\partial^2/\partial x^2 F_t = 1/c^2 \partial^2/\partial t^2 F_t \quad (71)$$

So the format for the vector potential equations are exactly the same, but the vector potentials ae not. As is typical, these two vector potentials are coupled by the speed of light c, where $F_t = 1/c^2 A_s$.

1.5 Bohm Aharonov Effect.

The Bohm Aharonov effect links the behavior of the electron not with electric and magnetic fields but with potentials. This effect was proposed in 1959 to show that the magnetic vector potentials is not just a theoretical construct, but has physical effects [4]. The first experiment was run in 1960 and showed a phase shift in electron interference fringes. Due to the effect that even a tiny residual magnetic field can have on an electron, the experiment was run again in 1986, this time with the latest material shielding technologies. Again, the same effect is seen. In the latter case, a superconductor was used and the magnetic flux is quantized, which quantized the phase shift seen in the electron interference pattern

$$\Delta\Phi = (e_0\, \Phi_m / \hbar) = \pi n \qquad (72)$$

In the experiment, two beams of electrons go around opposite sides of a solenoid to a screen where interference fringes can be imaged. The two different paths are identical. Outside the solenoid, the magnetic field is zero. When the magnetic field is turned on, the interference fringes of the electron shift, since the interaction of potentials results in a phase shift.

The same experiment was done with scalar potentials, where different potentials were applied to cylindrical metal tubes in each path. Again, the same result.

The Aharonov and Bohm effect is not limited to the magnetic vector potential. Aharonov and Bohm also describe a situation where particles are affected by regions where the electric field is zero, an electric Aharonov Bohm effect [4]. ***So, both the scalar potential and vector potentials affect particles without fields.***

In their paper, Aharonov and Bohm state that potentials are typically regarded as mathematical constructs and are considered to not have physical significance because equation of motions only involve fields. They, and other authors, after summarizing the results of the Aharonov and Bohm effect believe potentials to be more fundamental than fields [3,4,5].

1.6 Summary

The electrodynamic equations for 3D Vector Time have been derived based on the equations in ‘Concepts of Three-Dimensional Time in Electrodynamics [1]. The units for each step have been checked to make sure that they are consistent with three dimensions of Vector Time and motion in Vector Time.

Evaluating electrodynamics in terms of scalar and vector potentials show that the scalar wave equation is exactly the same for 4Space (Vector Space and Scalar Time) as it is for 4Time (Vector Time and Scalar Space). This suggests that the scalar potential is common to both the space and the time domain. The units for this scalar potential remain volts in both domains.

The vector potential has the same equation format as well, but the vector potential between 4Space and 4Time is different by a factor of $1/c^2$. In 4Space domain, this generates the magnetic field, and in 4Time it generates the electric field.

The symmetry of scalar and vector potentials and the scalar and vector terms for the 4Space and 4Time domain might be coincidence, but on the other hand, they might lead to some deeper understanding of the properties of time in the near future.

References

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Version

1. Original
2. Updated with reference to Electric Vector Potential, **F,** and magnetic scalar potential ψ. Changed the section in 3D time so that A_t is now F_f and Φ_t is changed to ψ_t so that it correlates better to the electric vector and magnetic scalar potential in electrodynamics. Add in reference four. Note: in other sources, the electric vector potential is denoted by vector C.
3. Included Aharonov-Bohm effect.
4. Nov 12,2016. Change email address.
5. Dec 11, 2017. Update terms to 4Space and 4Time.
6. Mar 11, 2018. Eliminated the subscript for all ε and μ. This changed the coef $1/c^2$ for $\mu_0\, \rho_{mt}$ as well as the coef c in front of $J_{es} / \varepsilon 0$.